Field," Ph.D. thesis, 1961, Massachusetts Institute of Technology, Cambridge, Mass.

[11] Dukowicz, J. K., Matthias, C. J., and de Leeuw, J. H., "The Flat-Plate Magnetohydrodynamic Boundary Layer in a Transverse Magnetic Field," *Canadian Aeronautics and Space Institute Transactions*, Vol. 1, No. 1, March 1968, pp. 45–51.

Unsymmetrical Bending of Shells of Revolution

J. J. Blech*
Ford Motor Company, Dearborn, Mich.

Based on the general first-order shell theory of Sanders, B. Budiansky and P. P. Radkowski[1] derived a set of four governing second-order differential equations. It has been pointed out by several authors[2–4] that for a computer analysis it is convenient to use as dependent variables those quantities which appear in the boundary conditions, thus reducing the shell field equations into a system of eight first-order differential equations. In such a formulation the computation of derivatives of wall properties is avoided. In case of the Sanders theory, using the same notation as in Ref. 1 a first-order system of equations is derived from the 17 field Eqs. (27–32) of Ref. 1 with the aid of Eqs. (5–7, 46, 49). This set of equations can be written in the matrix form

$$Z' = AZ + P \tag{1}$$

where the elements Z_i, P_i of the 8×1 column vectors Z, P, respectively, are given by

$$Z_1 = u_\xi,\quad Z_2 = u_\theta,\quad Z_3 = w,\quad Z_4 = \varphi_\xi,\quad Z_5 = t_\xi \\ Z_6 = \hat{t}_{\xi\theta},\quad Z_7 = \hat{f}_\xi,\quad Z_8 = m_\xi \tag{2a}$$

$$P_1 = -t_T/b,\quad P_2 = 0,\quad P_3 = 0,\quad P_4 = -m_T/d \\ P_5 = \gamma(1-\nu)t_T - p_\xi \\ P_6 = n/\rho(1-\nu)\,(t_T + \lambda^2\omega_\theta m_T) - p_\theta \\ P_7 = (1-\nu)\,(\omega_\theta t_T + \lambda^2 n^2/\rho^2 m_T) - p \\ P_8 = \gamma(1-\nu)m_T \tag{2b}$$

and the elements A_{ij} of the 8×8 matrix A are as follows

$$A_{11} = A_{41} = -\nu\gamma \tag{3a}$$

$$A_{12} = -A_{66} = -\nu n/\rho \tag{3b}$$

$$A_{13} = -A_{76} = -\omega_\xi - \nu\omega_\theta \tag{3c}$$

$$A_{16} = 1/b \tag{3d}$$

$$A_{21} = -A_{57} = (n/\rho)[1 - (\lambda^2 d/bc)(3\omega_\theta - \omega_\xi)\omega_\theta] \tag{3e}$$

$$A_{22} = -A_{56} = -A_{78} = -\tfrac{1}{2}A_{67} = \gamma \tag{3f}$$

$$A_{23} = \gamma A_{24} = -A_{77} = -\lambda^2 A_{87} = (\lambda^2 d/bc)(n/\rho)(3\omega_\theta - \omega_\xi) \tag{3g}$$

$$A_{27} = (2/bc)(1-\nu) \tag{3h}$$

$$A_{31} = -A_{58} = \omega_\xi \tag{3i}$$

$$A_{34} = -1 \tag{3j}$$

$$A_{42} = -(1/\lambda^2)A_{65} = -(\nu n/\rho)\omega_\theta \tag{3k}$$

$$A_{43} = -(1/\lambda^2)A_{75} = -\nu n^2/\rho^2 \tag{3l}$$

$$A_{45} = 1/d \tag{3m}$$

$$A_{51} = b(1-\nu^2)\left[\gamma^2 + \frac{2}{(1+\nu)c}\,\frac{\lambda^2 d}{b}\,\frac{n^2}{\rho^2}\,\omega_\theta^2\right] \tag{3n}$$

$$A_{52} = A_{61} = b(1-\nu^2)(n/\rho)\gamma \tag{3o}$$

$$A_{53} = A_{71} = b(1-\nu^2)\omega_\theta\gamma\left[1 - \frac{2}{(1+\nu)c}\,\frac{\lambda^2 d}{b}\,\frac{n^2}{\rho^2}\right] \tag{3p}$$

$$A_{54} = \lambda^2 A_{81} = \lambda^2 d(1-\nu^2)[2/(1+\nu)c](n^2/\rho^2)\omega_\theta \tag{3q}$$

$$A_{62} = b(1-\nu^2)(n^2/\rho^2)[1 + (\lambda^2 d/b)\omega_\theta^2] \tag{3r}$$

$$A_{64} = \lambda^2 A_{82} = \lambda^2 d(1-\nu^2)(n/\rho)\gamma\omega_\theta \tag{3s}$$

$$A_{73} = b(1-\nu^2)\left\{\omega_\theta^2 + \frac{\lambda^2 d}{b}\,\frac{n^2}{\rho^2}\left[\frac{n^2}{\rho^2} + \frac{2}{(1+\nu)c}\,\gamma^2\right]\right\} \tag{3t}$$

$$A_{74} = \lambda^2 A_{83} = \lambda^2 d(1-\nu^2)\,\frac{n^2}{\rho^2}\,\gamma\left[1 + \frac{2}{(1+\nu)c}\right] \tag{3u}$$

$$A_{84} = d(1-\nu^2)[\gamma^2 + \{2/(1+\nu)c\}n^2/\rho^2] \tag{3v}$$

$$A_{85} = -(1-\nu)\gamma \tag{3w}$$

$$A_{88} = 1/\lambda^2 \tag{3x}$$

where

$$c = 1 + (1/4)(\lambda^2 d/b)(3\omega_\theta - \omega_\xi)^2 \tag{4}$$

and all the other A_{ij}'s are equal to zero.

Equation (80) of Ref. 1 is replaced by

$$\sigma_\xi^{(n)} = \frac{\sigma_0}{(1-\nu^2)}\left(\frac{1}{b}\,t_\xi + \frac{1}{d}\,\frac{\zeta}{a}\,m_\xi\right) + \frac{t_T}{b} + \frac{\zeta}{a}\,\frac{m_T}{d} - \frac{E\alpha T^{(n)}}{(1-\nu)} \tag{5a}$$

$$\sigma_\theta^{(n)} = \sigma_0\left[\gamma u_\xi + \frac{n}{\rho}\left(1 + \omega_\theta\,\frac{\zeta}{a}\right)u_\theta + \left(\omega_\theta + \frac{\zeta}{a}\,\frac{n^2}{\rho^2}\right)\times w + \gamma\,\frac{\zeta}{a}\,\varphi_\xi + \frac{\nu}{(1-\nu^2)}\left(\frac{1}{b}\,t_\xi + \frac{1}{d}\,\frac{\zeta}{a}\,m_\xi\right)\right] + \nu\,\frac{t_T}{b} + \nu\,\frac{\zeta}{a}\,\frac{m_T}{d} - \frac{E\alpha T^{(n)}}{(1-\nu)} \tag{5b}$$

$$\sigma_{\xi\theta}^{(n)} = \frac{\sigma_0}{(1+\nu)}\left\{\left[cA_{24} - \frac{n}{\rho}\,\frac{\zeta}{a}\right](-\omega_\theta u_\xi + \gamma w + \varphi_\xi) + \frac{1}{bc(1-\nu)}\left[1 + (3\omega_\theta - \omega_\xi)\,\frac{\zeta}{a}\right]\hat{t}_{\xi\theta}\right\} \tag{5c}$$

By partitioning the column vector Z into two 4×1 column vectors $y = (u_\xi, u_\theta, w, \varphi_\xi)$ and $z = (t_\xi, \hat{t}_{\xi\theta}, \hat{f}_\xi, m_\xi)$, the branch point transition relations (B1), (B2) of Ref. 1 are simplified to

$$y^{\mathrm{III}} = \psi^{\mathrm{II}}y^{\mathrm{II}} = \psi^{\mathrm{I}}y^{\mathrm{I}} \tag{6a}$$

$$z^{\mathrm{III}} = \psi^{\mathrm{II}}z^{\mathrm{II}} + \psi^{\mathrm{I}}z^{\mathrm{I}} \tag{6b}$$

where the matrix ψ is given by Eq. (57) of Ref. 1.

The boundary conditions and discontinuity condition equations that are used in the present formulation are similar to Eqs. (47, 55, and 56) of Ref. 1, respectively.

References

[1] Budiansky, B. and Radkowski, P. P., "Numerical Analysis of Unsymmetrical Bending of Shells of Revolution," *AIAA Journal*, Vol. 1, No. 8, Aug. 1963, pp. 1833–1842.

[2] Cohen, G. A., "Computer Analysis of Asymmetrical Deformation of Orthotropic Shells of Revolution," *AIAA Journal*, Vol. 2, No. 5, May 1964, pp. 932–934.

[3] Goldberg, J. E., "Computer Analysis of Shells," 70th Anniversary Symposium on the Theory of Shells, Univ. of Houston, April 1966.

[4] Kalnins, A., "Analysis of Shells of Revolution Subjected to Symmetrical and Nonsymmetrical Loads," *Journal of Applied Mechanics*, Vol. 31, Sept. 1964, pp. 467–476.

Received September 15, 1969.

* Turbine Research Specialist.